Nonlinear Electro-Mechanical Modeling of MEMS Switches

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Abstract— **We present an accurate model of the switching mechanism of MEMS switches. The model is based on a electro-mechanical analysis which takes into account the varying force and damping versus position (time). The model also calculates the switching current taking into account both the capacitance change and the voltage change versus time. The model accurately predicts the switching time, the switching current, the velocity versus position (and time) of the MEMS bridge, and the energy consumed in the switching process. It is found that the current can be very large and the total switching energy is larger than predicted by simple models due to the damping underneath the MEMS bridge.**

I. Introduction

MICRO -mechanical series and shunt switches have shown some very impressive results (insertion loss and isolation) from 0.1 to 100 GHz [1], [2], [3], and have been employed in low-loss phase shifters at 10 GHz, 35 GHz and 40-100 GHz. The electrical performance of MEMS switches are now well understood. A MEMS DC-contact series switch is accurately modeled by a capacitance in the up-state position, and a resistance in the down-state position. A MEMS shunt capacitive switch is accurately modeled by a capacitance in the up-state position, and a CLR model in the down-state position [1], [4]. The mechanical analysis of MEMS switches has not followed a parallel approach, and the goal of this paper is to introduce an accurate electro-mechanical model which predicts virtually everything about the switching mechanism of RF MEMS switches (with the possible exception of stiction and charge trapping).

II. Simple Mechanical Analysis

The MEMS switches are modeled using a simple fixed-fixed beam or a cantilever design. The pull-down voltage is derived from *static* calculation and is:

$$V_p = \sqrt{\frac{8k}{27\epsilon_o W w} g^3} \tag{1}$$

where k is the spring constant (or modal stiffness) of the MEMS bridge (or cantilever), g is the height of the MEMS bridge over the pull down electrode, w is the width of the MEMS bridge, and W is the size of the pull down electrode. The spring constant is dependent on the bridge geometry

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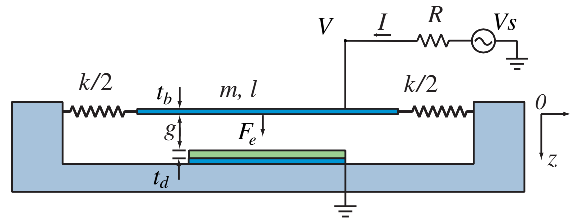


Fig. 1. Coordinate system and simplified mechanical model of a MEMS fixed-fixed beam switch.

and, for a fixed-fixed beam design with a force distributed over the center third of the bridge length, k is given by:

$$k = \frac{32Ewt^3}{l^3}\left(\frac{27}{49}\right) + \frac{8\sigma(1-\nu)wt}{l}\left(\frac{27}{49}\right) \tag{2}$$

where E is the Young's modulus, ν is the Poisson's ratio, σ is the residual stress in the fixed-fixed beam, l, w, and t are the length, width and thickness of the beam, respectively. Similar equations can be derived for cantilever switches [5]. The switching time is derived using the Newtonian dynamic equation of motion and neglecting the damping underneath the MEMS switch (or cantilever). Also, the applied force is assumed to be constant and is given by the electrostatic force due to a voltage V on the MEMS bridge, and is [6]:

$$F_e = \frac{1}{2}\frac{\epsilon_o w W V^2}{g^2} \tag{3}$$

The dynamic equation of motion becomes:

$$m\frac{d^2z}{dt^2} + kz = F = \frac{1}{2}\frac{\epsilon_o w W V^2}{g^2} \tag{4}$$

where m is the mass of the bridge (or modal mass), and z is the displacement from the up-state position. The initial conditions are $z = 0$ and $dz/dt = 0$ at $t = 0$ (switch is at rest), and the switching time is calculated for $z = g$, and is [6]:

$$t_s = \frac{V_p}{\omega_o V_s}\sqrt{\frac{27}{2}} \tag{5}$$

where $\omega_o = \sqrt{\frac{k}{m}}$ is the mechanical resonant frequency of the bridge, V_s is the source (applied) voltage, and V_p is the pull-down voltage given in Equation 1. The switching time is dependent on the applied voltage. The higher the applied voltage, the faster the switch. This analysis is accurate if the applied voltage is larger than 2-3V_p.

The energy consumed in the switching process can be calculated as the sum of the electric and mechanical energy in the MEMS bridge. The mechanical energy is the energy stored in the bridge spring and is given by $E_m = kg^2/2$. The electrical energy is the energy stored in the MEMS capacitor, and is $E_e = C_d V_s^2/2$ (assuming $C_d \gg C_u$). Using these values, the energy required for a MEMS bridge with $k = 6$ N/m, $g = 2.5$ μm, $C_d = 2$ pF and $V_s = 50$ V is $E = E_e + E_m = 2.5$ nJ and $E_e \gg E_m$. We will see later that this estimate of the switching energy is inaccurate, and that the damping mechanism must be included in the switching energy computations.

III. Mechanical Analysis with Intermediate Accuracy

An intermediate mechanical analysis takes into account the varying force versus position (or time) as the MEMS bridge is being pulled down to the bottom electrode. The analysis also takes into account the damping factor of the air layer underneath the MEMS bridge. The dynamic equation of motion becomes:

$$m\frac{d^2z}{dt^2} + b\frac{dz}{dt} + kz = F_e + F_c \tag{6}$$

$$F_e = \frac{1}{2}\frac{\epsilon_o w W V^2}{(g + t_d/\epsilon_r - z)^2} \tag{7}$$

where t_d is the dielectric thickness with a dielectric constant of ϵ_r, b is the damping coefficient and is dominated by the squeeze-film damping under the bridge, F_c is the contact force at the dielectric/metal interface. For two parallel plates, b is given by [7]:

$$b = \frac{k}{\omega_o Q} \simeq \sqrt{2}\mu_{air} l \left(\frac{w}{g}\right)^3 \tag{8}$$

where ω_o is the natural resonant frequency of the switch, Q is the quality factor of the oscillating bridge, and μ_{air} is the viscosity of air ($\approx 1.8 \times 10^{-5}$kg/m^3). The damping coefficient (b) for an arbitrary switch can be calculated from Eq. (8) and direct measurements of the spring constant (using an atomic force microscope) and the quality factor (extracted from the small displacement frequency response).

Equation 6 is a non-linear differential equation, and its solution can be obtained using a non-linear solver such as Mathematica [8]. Again, the boundary conditions are $z = 0$ and $dz/dt = 0$ at $t = 0$. Figure 2 presents the time-domain solution for a MEMS gold bridge with $t = 0.8$ μm, $w = 60$ μm, $l = 300$ μm, $W = 100$ μm, $k = 6$ N/m, $g = 2.5$ μm, $V_p = 25$ V and $V_s = 35$, 50 V. The solution is valid until the contact is achieved, and this is for $x = t_d$. The switching time is very similar to the simple mechanical solution presented above for $V_s = 50$ V. The MEMS bridge speed (dz/dt) is 2-3 m/s, for $V_s = 35$-50 V, just before hitting the bottom electrode.

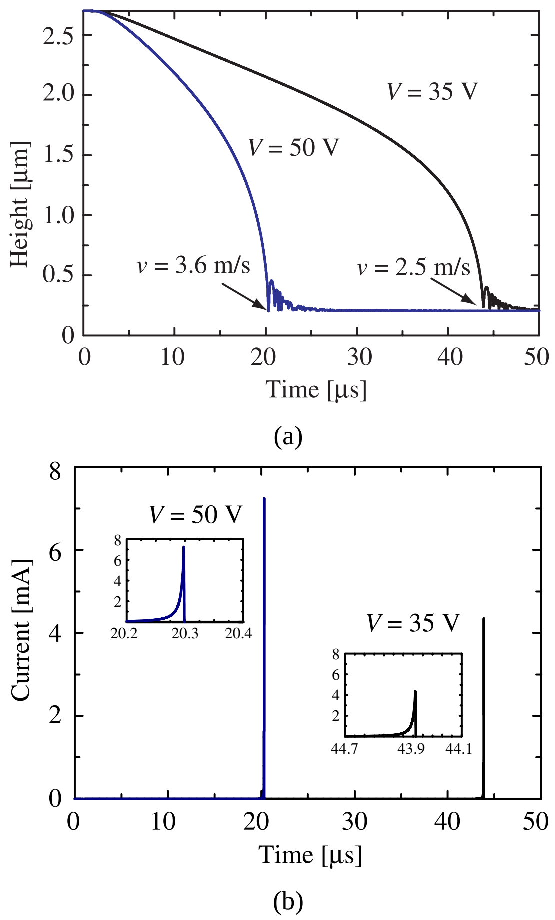


Fig. 2. Simulated (a) bridge height and (b) current versus time for a step voltage of 35 V and 50 V.

The solution of the MEMS bridge position versus time can be used to extract the switching current. The current is given by:

$$I = \frac{dq}{dt} = C\frac{dV}{dt} + V\frac{dC}{dt} \tag{9}$$

$$C = \frac{\epsilon_o w W}{g + t_d/\epsilon_r - z} \tag{10}$$

and z versus t is given in Figure 2a. Notice that the peak switching current occurs just near the end of the switching cycle where the bridge speed is the highest. The switching current is shown in Figure 2b. for the cases outlined above, and is 4 mA ($V_s = 35$ V) to 8 mA ($V_s = 50$ V), which is significant. The energy consumed in the switching process can be calculated as:

$$E = V \int i dt = E_c + E_m + E_k + E_d \qquad (11)$$

where V_s is the source voltage (30 or 50 V), $i(t)$ is the switching current, E_e, E_m are the mechanical and electrical energy defined above, $E_k = \frac{1}{2}mv^2$ and is the kinetic energy of the MEMS bridge, and E_d is the energy dissipated in the damping mechanism. For $V_s = 30$ V and 50 V, the switching energy is calculated to be 1.98 nJ and 4.1 nJ, respectively. The energy stored in the capacitor, E_c is 1.0 nJ and 2.1 nJ for 35 V and 50 V, respectively, showing that the kinetic energy and the damping energy account for 50% of the total switching energy at the point of contact.

IV. Accurate Electro-Mechanical Analysis

The accurate electro-mechanical analysis follows the same approach as the intermediate analysis above, with the following changes:

- The damping factor is allowed to change versus position (time)
- A contact force, F_c is taken in the analysis which simulates the attractive and repulsive forces between the metal bridge and the dielectric layer.
- The switching current versus position (or time) is used with the bias resistor (20 kΩ) to calculate the voltage drop from the power supply to the MEMS bridge, and to result in a true value of the bias voltage versus position (or time) applied to the MEMS bridge.

The above analysis results in two simultaneous non-linear differential equations when included in the dynamic equation of motion. Again, they are solved using Mathematica.

$$m\frac{d^2z}{dt^2} + b\left(1.2 - \frac{z}{g_o}\right)^{-\frac{3}{2}}\frac{dz}{dt} + kz = F_e + F_c \qquad (12)$$

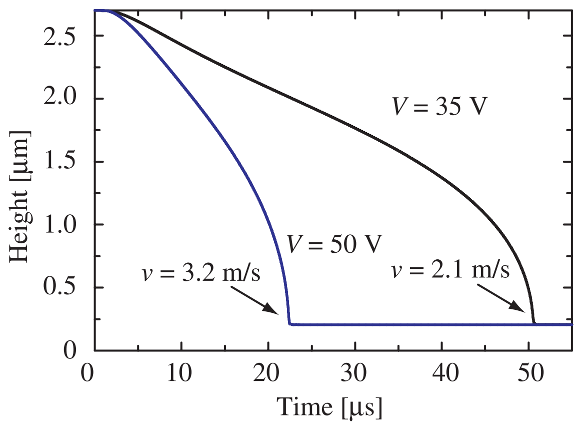
$$V = V_s - i(t)R = V_s - \left(C\frac{dV}{dt} + V\frac{dC}{dt}\right)R \qquad (13)$$

The solutions for the case outlined in Section III are shown in Figure 3 for $V_s = 35$ V and $V_s = 50$ V. Also shown are the switching currents and the voltage on the MEMS switch versus time. Note that the applied voltage decreases before the end of the switching cycle due to the voltage drop in the 20 kΩ resistor.

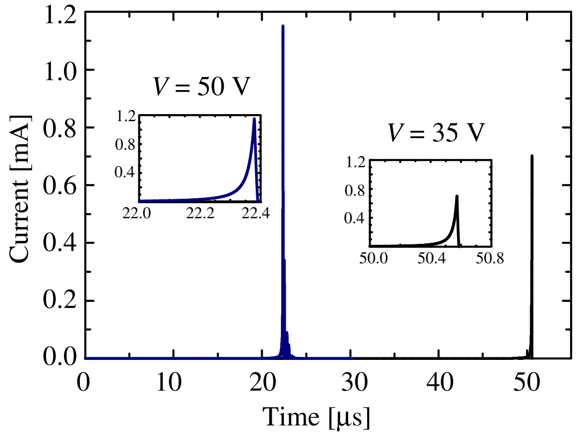
The energy delivered by the source and consumed in the switching process can be calculated as:

$$E = V_s \int i(t)dt = E_e + E_m + E_R + E_k + E_d \qquad (14)$$

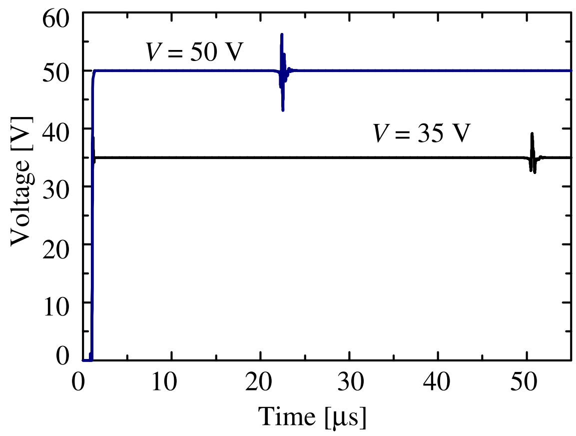
where $E_R = R\int i(t)^2 dt$ is the energy dissipated in the resistor. For $V_s = 35$ V and 50 V, the total energy used is



(a)



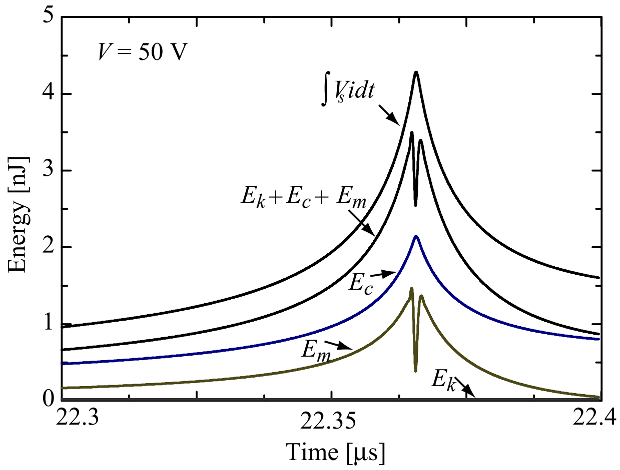
(b)



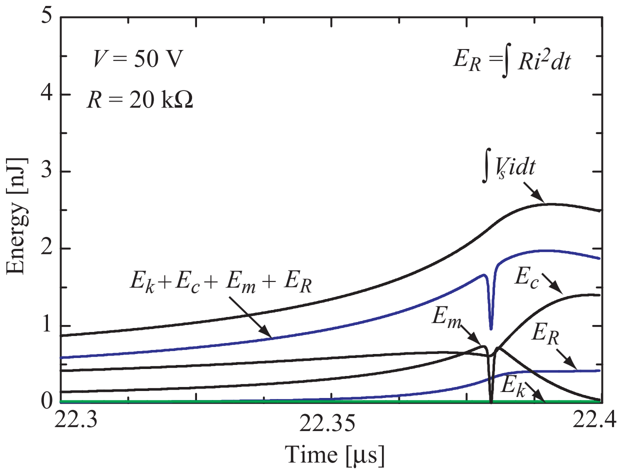
(c)

Fig. 3. Simulated (a) bridge height and (b) current and (c) capacitor voltage versus time for a step voltage of 35 V and 50 V.

1.19 nJ and 2.20 nJ, respectively (Fig. 4. The reduction in the bridge speed is due to the bias resistor which results in a reduced voltage on the MEMS switch at the end of the switching cycle.



(a)



(b)

Fig. 4. Simulated energy versus time for (a) no bias resistance and (b) a bias resistance of 20 kΩ.

Table I presents the energy balance at the point of contact in the switching process for the MEMS bridge described above. The potential energy stored in the stretched membrane, E_k, is very small (19 pJ) and is not included in Table I. Also, the damping component contributes to the energy balance and accounts for 16-25% of the total switching energy. The dominant effect of an increased damping factor (conversely a decreased Q) is the dramatic decrease in the switching time for $V_s = 1.2\text{-}1.5V_p$.

The presence of the 20 kΩ bias lines reduces the kinetic energy (and velocity) of the MEMS switch at the point of contact by causing the voltage across the switch to drop when there is a rapid change in capacitance, as occurs just before the membrane contacts the dielectric layer.

TABLE I

ENERGY BALANCE FOR A MEMS GOLD BRIDGE WITH $t = 0.8$ μM, $w = 60$ μM, $l = 300$ μM, $W = 100$ μM, $k = 6$ N/M, $g = 2.5$ μM, $Q = 0.3$, $V_p = 25$ V AND $V_s = 35$, 50 V. ENERGY IS IN NJ.

V_s [V]	R [Ω]	E	E_R	E_m	E_c	E_d
35	0	1.97	0	0.59	0.99	0.37
50	0	4.10	0	1.35	2.05	0.68
35	20 k	1.19	0.13	0.34	0.37	0.73
50	20 k	2.20	0.28	0.74	0.62	0.54

V. CONCLUSION

This paper presents an accurate model of the switching energy in MEMS electrostatic switches. The switching energy is quite low (2-4 nJ). However, the speed of the switch at impact is high (2-4 m/s) and this can affect the switch reliability. Also, the currents involved are substantial and should be taken into account in the design of the DC switching networks. The presence of high impedance bias lines slows down the switch, and reduces the total and kinetic energy of the switch at the moment of contact.

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